* **Dimensions:** * Seven base quantities referred as Seven dimensions of physical world.

* Represented by using square brackets.

Base quantity	Dimensional formula
1. Mass	[M]
2. length	[L]
3. time	[T]
4. current	[A] or [I]
5. Temperature	[K] or [θ]
6. Luminous intensity	[cd]
7. Amount of Substance	[mol]

Definition: The powers to which the base quantities are raised to represent the quantity are called dimensions of that quantity.

Ex: Speed = $\frac{\text{distance}}{\text{time}}$ ⇒ $v = \frac{d}{t}$

$[v] = \frac{[d]}{[t]} = \frac{[L]}{[T]} = [L T^{-1}]$

Dimensions of speed in mass → 0
length → 1
time → -1

$[v] = [M^0 L^1 T^{-1}]$

Dimensions of speed $\longrightarrow 0, 1, -1$

Dimensional formula of speed $\longrightarrow [M^0 L^1 T^{-1}]$

Dimensional equation of speed $\longrightarrow [V] = [M^0 L^1 T^{-1}]$

* 1. $[\text{Speed}] = [\text{Velocity}] = [L T^{-1}]$

2. $[\text{distance}] = [\text{Displacement}] = [L]$

3. momentum $\quad p = \text{mass} \times \text{velocity}$

$$p = mv$$

$$[p] = [m][v]$$

$$[p] = [M L T^{-1}]$$

4. acceleration $\quad a = \dfrac{\text{change in velocity}}{\text{time}}$

$$[a] = \frac{[v]}{[t]}$$

$$= \left[\frac{L T^{-1}}{T}\right]$$

$$= [L T^{-2}]$$

5. Force $\quad F = \text{mass} \times \text{acceleration}$

$$[F] = [m][a]$$

$$= [M L T^{-2}]$$

Notes:

* Area = length × breadth

$[A] = L^2$

* Volume = length × breadth × height

$[V] = L^3$

* Density = $\dfrac{\text{mass}}{\text{volume}}$

$$[\rho] = \frac{[m]}{[V]}$$

$$= \left[\frac{M}{L^3}\right]$$

$$= [M L^{-3}]$$

6. Impulse J = change in momentum (or) Force x time

$[J] = [P]$ $\quad P \rightarrow$ momentum

$= [m\,v]$ $\quad m \rightarrow$ mass, $v \rightarrow$ velocity

$= [M L T^{-1}]$

[impulse] = [momentum] = $[M L T^{-1}]$

7. Workdone W = Force x displacemt.

$[W] = [F][S]$ $\quad F \rightarrow$ Force

$= [M L T^{-2}][L]$ $\quad S \rightarrow$ displacement

$= [M L^2 T^{-2}]$

$[\text{Energy}] = [K\cdot E] = [\frac{1}{2} m v^2]$ $\quad K\cdot E$ = Kinetic energy

(or)

$[P\cdot E] = [m][v]^2$ $\quad P\cdot E$ = potential energy.

$= [M][L T^{-1}]^2$ $\quad m \rightarrow$ mass

$= [M L^2 T^{-2}]$ $\quad v \rightarrow$ speed

Torque τ = Force x $\perp$ar distance

$[\tau] = [F] \times [r]$

$= [M L T^{-2}][L]$

$= [M L^2 T^{-2}]$

$\because$ [Workdone] = [Energy] = [Torque] = $[ML^2T^{-2}]$

8. power $\quad P = \frac{\text{Work}}{\text{time}}$

$$[P] = \frac{[W]}{[t]}$$

$$= \frac{[ML^2T^{-2}]}{[T]}$$

$$= [ML^2T^{-3}]$$

9. Intensity (or) solar constant $\quad I = \frac{\text{Energy}}{\text{Area} \times \text{time}}$

$$[I] = \frac{[E]}{[A][t]}$$

$$= \frac{ML^2T^{-2}}{[L^2][T]}$$

$$= [MT^{-3}]$$

10. pressure (or) stress $\quad P = \frac{\text{Force}}{\text{Area}}$

$$[P] = \frac{[F]}{[A]}$$

$$= \frac{[MLT^{-2}]}{[L^2]}$$

$$= [ML^{-1}T^{-2}]$$

[same formula for elastic modulus $[E] = [ML^{-1}T^{-2}]$

$E = \frac{\text{Stress}}{\text{Strain}}$]

*11. Strain = $\dfrac{\text{change in length (or) Area (or) volume}}{\text{initial length (or) Area (or) volume}}$

$$= \frac{\Delta L}{L} \text{ (or) } \frac{\Delta A}{A} \text{ (or) } \frac{\Delta V}{V}$$

$\therefore$ [Strain] = $[M^0 L^0 T^0]$

*12. Coefficient of elasticity E = $\dfrac{\text{Stress}}{\text{strain}}$

(or)

Elastic modulus

$$[E] = \frac{[\text{Stress}]}{[\text{Strain}]}$$

$$[E] = \frac{[M L^{-1} T^{-2}]}{[M^0 L^0 T^0]}$$

$$[E] = [M L^{-1} T^{-2}]$$

$\therefore$ [Pressure] = [Stress] = [Elastic modulus]

$$= [M L^{-1} T^{-2}]$$

13. Energy density = $\dfrac{\text{Energy}}{\text{volume}}$

$$[U] = \frac{[E]}{[V]} = \frac{[M L^2 T^{-2}]}{[L^3]} = [M L^{-1} T^{-2}]$$

Energy density of electric field $U_E = \frac{1}{2} \epsilon_0 E^2$ [E → electric field, ϵ_0 → permitivity of free space]

Energy density of magnetic field $U_B = \frac{B^2}{2\mu_0}$

$B \rightarrow$ magnetic field

$\mu_0 \rightarrow$ permiability

$\therefore$ [Energy density U] $= [U_E] = [U_B] = [M L^{-1} T^{-2}]$

14. coefficient of viscocity $= \frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$

$$[\eta] = \left[\frac{F d}{A V}\right]$$

$$= \left[\frac{M L T^{-2} L}{L^2 L T^{-1}}\right]$$

$$= \left[\frac{M L^2 T^{-2}}{L^3 T^{-1}}\right]$$

$$= [M L^{-1} T^{-1}]$$

15. surfacetension $[T] = \left[\frac{\text{Force}}{\text{length}}\right]$

$$= \left[\frac{F}{L}\right]$$

$$= \left[\frac{M L T^{-2}}{L}\right]$$

$$= [M T^{-2}]$$

16. velocity gradient $= \frac{\text{velocity}}{\text{distance}}$

(or)

frequency $f = \frac{[V]}{[L]}$

$= \left[\frac{L T^{-1}}{L}\right]$

$= [T^{-1}]$

pressure gradient $= \frac{\text{pressure}}{\text{length}}$

$\therefore = \left[\frac{M L^{-1} T^{-2}}{L}\right]$

$= [M L^{-2} T^{-2}]$

[velocity gradient] = [frequency] $= [T^{-1}]$

17. * Angle

plane angle $= \frac{\text{Arc length}}{\text{radius}}$

[plane angle] $= [M^0 L^0 T^0]$

solid angle $= \frac{\text{surface area}}{\text{square of radius}}$

[solid angle] $= [M^0 L^0 T^0]$

[Angle] = [plane angle] = [solid angle] $= [M^0 L^0 T^0]$

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18. Angular velocity $\omega = \frac{\text{Angle}}{\text{time}}$

$$[\omega] = \left[\frac{M^0 L^0 T^0}{T}\right]$$

$$[\omega] = [T^{-1}]$$

$\therefore$ $[\omega]$ = [frequency] = [velocity gradient] = $[T^{-1}]$

19. Angular acceleration $\alpha = \frac{\text{Angular velocity}}{\text{time}}$

$$[\alpha] = \frac{[\omega]}{[t]} = \frac{[T^{-1}]}{[T]} = [T^{-2}]$$

20. Angular momentum L = mass x velocity x radius

(same for plank's constant)

$$[L] = [mvr]$$

$$= [M L T^{-1} L]$$

$$= [M L^2 T^{-1}]$$

* speed of light $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/s

$$[c] = \left[\frac{1}{\sqrt{\mu_0 \epsilon_0}}\right] \text{(or)} \left[\frac{1}{\mu_0 \epsilon_0}\right]^{1/2} \text{(or)} [\mu_0 \epsilon_0]^{-1/2} = [L T^{-1}]$$

* plank's constant $h = \frac{\text{Energy}}{\text{frequency}}$

$$[h] = \left[\frac{E}{\nu}\right]$$

$$[h] = \left[\frac{M L^2 T^{-2}}{T^{-1}}\right]$$

$$[h] = [M L^2 T^{-1}]$$

* universal gravitational constants $G = \frac{F r^2}{m_1 m_2}$

$$[G] = \left[\frac{M L T^{-2} L^2}{M^2}\right]$$

$$[G] = [M^{-1} L^3 T^{-2}]$$

* $F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \Rightarrow \frac{e^2}{4\pi\epsilon_0} = F r^2$

$$\left[\frac{e^2}{4\pi\epsilon_0}\right] = [F r^2]$$

$$= [M L T^{-2} L^2]$$

$$= [M L^3 T^{-2}]$$

* universal gas constant $R = \frac{PV}{nT}$

$$[R] = \left[\frac{ML^{-1}T^{-2}L^{3}}{mol\ K}\right]$$

$$= [ML^{2}T^{-2}mol^{-1}K^{-1}]$$

* Boltzman's constant $K = \frac{\text{Energy}}{\text{temperature}}$

$$= \left[\frac{ML^{2}T^{-2}}{K}\right]$$

$$= [ML^{2}T^{-2}K^{-1}]$$

Dimensionless quantities: plane angle, solid angle, T-ratios ($\sin\theta, \cos\theta, \tan\theta \ldots$), Relative density, Strain, refracive index ($\mu = \frac{c}{v}$), pure numbers, trigonometric, logarithmic, exponential functions

Ex:

$y = A\sin(\omega t)$ $\quad y \rightarrow$ distance

$[\omega t] = [M^{0}L^{0}T^{0}]$ $\quad t \rightarrow$ time

$y = Ae^{(ct)}$, $y = A\log(\omega t)$

$[ct] = [M^{0}L^{0}T^{0}]$ $\quad [\omega t] = [M^{0}L^{0}T^{0}]$

principle of homogeniety:

* Any two physical quantities which are going to be added, subtracted or equated must have same dimensions.

Ex: (i) $a + b \longrightarrow$ valid

$[a] = [b]$

* All terms in a given equation should have same dimensions.

(ii) $c - d \longrightarrow$ valid

$[c] = [d]$

(iii) $P = Q \longrightarrow$ valid

$[P] = [Q]$

(iv) $ax + by = cp + dq \longrightarrow$ valid

For L.H.S $[ax] = [by] = [cp] = [dq]$ R.H.S

Ex: $y = At + Bt^2$

$y \rightarrow$ distance

$t \rightarrow$ time

$[y] = [At] = [Bt^2]$

$[y] = [At]$

$[A] = \frac{[y]}{[t]} = \frac{[L]}{[T]}$

$= [LT^{-1}]$

$[y] = [Bt^2]$

$[B] = \frac{[y]}{[t^2]} = \frac{[L]}{[T^2]}$

$= [LT^{-2}]$

Ex: (i) $y = A \sin(\omega t)$ $y \to$ distance

$t \to$ time

What is $[A] = ?$ $[\omega] = ?$

Trigonometric functions are dimensionless

$$[\omega t] = [M^0 L^0 T^0]$$

$$[\omega] = \left[\frac{M^0 L^0 T^0}{T}\right]$$

$$[\omega] = [T^{-1}] \checkmark$$

and $[y] = [A]$

$$\therefore [L] = [A] \checkmark$$

(ii) $y = A \log(Bx + Ct)$ x, y - distance

t - time

$[A] = [y]$ $[Bx + Ct] = [M^0 L^0 T^0]$ $[A] = ?$ $[B] = ?$ $[C] = ?$

$= [L]$

$[Bx] = [M^0 L^0 T^0]$ $[Ct] = [M^0 L^0 T^0]$

$[Bx] = [1]$ $[Ct] = [1]$

$[B] = \left[\frac{1}{x}\right]$ $[C] = \left[\frac{1}{t}\right]$

$= \left[\frac{1}{L}\right]$ $[C] = \left[\frac{1}{T}\right]$

$= [L^{-1}]$ $[C] = [T^{-1}]$

(iii) $y = A e^{(Bt)}$

$y \rightarrow$ distance

$t \rightarrow$ time

$[A] = ?$ $[B] = ?$

$\because$ exponential functions are dimension less

$$[Bt] = [M^0 L^0 T^0]$$

$$[Bt] = [1]$$

$$[B] = \left[\frac{1}{t}\right]$$

$$= \left[\frac{1}{T}\right]$$

$$= [T^{-1}]$$

Applications:

1. **Checking the dimensional consistancy of equations:** We can check any equation

Ex: wheather it is dimensionally correct equation or not.

Ex: We know that the second equation of motion

$$S = ut + \frac{1}{2}at^2$$

$S \rightarrow$ displacement

$u \rightarrow$ initial speed

$a \rightarrow$ acceleration

$t \rightarrow$ time.

According to principle of homogeniety

$$[L.H.S] = [R.H.S]$$

Let us check it

$$[S] = [L]$$

$$[ut] = [LT^{-1}T] = [L]$$

$$[at^2] = [LT^{-2}T^2] = [L]$$

$\therefore [S] = [ut] = [at^2]$

$\therefore S = ut + \frac{1}{2}at^2$ is dimensionally correct.

We know that it is mathematically correct also.

If $S = ut + \frac{1}{9} a t^2$

we know that it is wrong mathematically (or) physically

Let me check the dimensionality

$[L \cdot H \cdot S] = [R \cdot H \cdot S]$ is valid or not

$[S] = [L]$

$[ut] = [LT^{-1}T] = [L]$

$[at^2] = [LT^{-2}T^{2}] = [L]$

* $S = ut + \frac{1}{9} at^2$ equation is dimesionally correct but mathematically (or) physically wrong.

Important note: * A dimensionally correct equation need not to be correct mathematically

but

* A mathematically correct equation must be dimensionally correct.

2. Deducing relation among various physical quantities:

Ex: A body is dropped from a height 'h', the time taken to reach ground is 't'

Q. Express dimensional formula of time "t" in terms of height "h", acceleration due to gravity "g"

$$t \propto h^x g^y$$

$$t = K h^x g^y$$

According to principle of homogeniety.

$$[t] = [h^x g^y]$$

$$[t] = [h]^x [g]^y$$

$$[T] = [L]^x [LT^{-2}]^y$$

$$[M^0 L^0 T^1] = [L^x][L^y T^{-2y}]$$

$$[M^0 L^0 T^1] = [L^{x+y} T^{-2y}]$$

$$x+y=0 \quad , \quad -2y = 1$$

$$x = -y \qquad y = -1/2$$

$$= -(-1/2) = 1/2$$

$$\therefore \quad t = K\, h^{1/2}\, g^{-1/2}$$

$$t = K \sqrt{h}\, \frac{1}{\sqrt{g}}$$

$$\therefore \quad t = K\sqrt{\frac{h}{g}}$$

mathematically $K = \sqrt{2}$ (experimentally found value).

Q. Express base quantities interms of derived quantities:

Ex: Distance travelled by the body depends on Force F, velocity v, time t

express dimensional formula of distance (or) length interms of F, v, t.

$$\therefore \quad L \propto F^{x}\, v^{y}\, t^{z}$$

$$[L] \propto [F]^{x}\, [v]^{y}\, [t]^{z}$$

$$[L] \propto [MLT^{-2}]^{x}\, [LT^{-1}]^{y}\, [T]^{z}$$